A mathematical model and a method to measure the instantaneous power output of vehicular driving wheels

Wei Zhao, Zhi-zhong Li, and Hai-tao Zhang

Abstract: The internal resistance of a dynamometer is regarded as a constant force when using traditional test methods, which means that the measurement values for the instantaneous power output of the vehicle driving wheel are underestimated. In this study, we aimed to resolve this issue. Thus, we constructed a system for measuring drum movement and propose a method for measuring the instantaneous power output of a vehicle's driving wheel. Further, we created a mathematical model of the instantaneous power output from a vehicle's driving wheel. Comparing the results from our model with our experimental results, we found that the actual power output of the driving wheel was reflected more accurately by our proposed method, and that the accuracy of the measurement results was not affected by the vehicle's speed. The output torque of the vehicle's driving wheel calculated using our proposed model was closer to the running resistance of the vehicle. Thus, our model facilitates accurate no-dismounting, rapid measurement of the power output of a driving wheel, and accurate simulation of road running resistance using a dynamometer.

Key words: driving wheel, instantaneous power output, detection, equivalent moment of inertia.

Résumé : Afin de résoudre le problème de la résistance interne du dynamomètre considérée comme une force constante par la méthode d'essai traditionnelle, ce qui rend la précision de mesure de la puissance de sortie instantanée de la roue motrice du véhicule insuffisante, un système de mesure du mouvement du tambour a été réalisé, une méthode de mesure de la puissance de sortie instantanée de la roue motrice du véhicule a été proposée et un modèle mathématique de la puissance de sortie instantanée de la roue motrice du véhicule a été établi. L'analyse du modèle et les résultats expérimentaux ont montré que la puissance de sortie réelle de la roue motrice était reflétée plus précisément par la méthode proposée, et que la précision des résultats de mesure n'était pas affectée par la vitesse du véhicule. Le couple de sortie de la roue motrice du véhicule calculé par le modèle proposé était plus proche de la résistance à l'avancement du véhicule, ce qui a fourni un soutien technique pour la détection rapide et sans démontage de la puissance de sortie de la roue motrice du véhicule et la simulation précise de la résistance à l'avancement du véhicule par le dynamomètre. [Traduit par la Rédaction]

Mots-clés : roue motrice, puissance de sortie instantanée, détection, moment d'inertie équivalent.

1. Introduction

The traditional method for testing vehicle driving performance involves conducting road driving tests to establish a driving resistance model for analysis and research. This method is affected by external environmental factors such as air pressure, temperature, and humidity, which complicate the detection process (Páscoal et al. 2012). Chassis dynamometers, as test platforms for overall vehicle performance, have been widely used to collect data on the dynamic performance of vehicles, braking, fuel economy, emissions, and other

variables (Nakashima et al. 2017; Chen et al. 2018; Triantafyllopoulos et al. 2019). A chassis dynamometer mainly comprises a drum, a simulated resistance-loading device, a rotational speed and torque measuring device, and a control and display device (Chelia et al. 2011). The power output of the driving wheel is an important index with which to measure the vehicle's dynamic performance. With the traditional method for testing the power output of the driving wheel with a chassis dynamometer, the driving wheel of the vehicle is placed on the drum, the output torque of the engine passes

Received 5 January 2021. Accepted 6 July 2021.

W. Zhao, Z.-z. Li, and H.-t. Zhang. National Defense Engineering College, Peoples Liberation Army Engineering University, Nanjing, Jiangsu 21007, China.

Corresponding author: Wei Zhao (email: dnallzw@163.com).

© 2021 The Authors. This work is licensed under a [Creative Commons Attribution 4.0 International License](https://creativecommons.org/licenses/by/4.0/) (CC BY 4.0), which permits unrestricted use, distribution, and reproduction in any medium, provided that the original author(s) and source are credited.

through the vehicle's transmission system to the driving wheel (which causes the drum and transmission device of the dynamometer to rotate), and a braking torque that is opposite to the driving torque on the rotor of the dynamometer is generated by the eddy current, which is equal to the driving torque of the driving wheel to the drum. By measuring the electromagnetic torque, the driving torque of the driving wheel on the drum can be obtained, and the power output of the driving wheel can be obtained by multiplying the driving torque by the drum speed (Sun et al. 2018).

However, in this traditional method for measuring the power output of the driving wheel, the measured power does not include the power loss due to the rolling resistance between the driving wheel and the drum and the power loss due to friction inside the drum chassis dynamometer during the test, owing to the friction loss between the vehicle's tires and the drum of the dynamometer as well as the friction loss due to the dynamometer transmission system. Therefore, the driving power of a vehicle cannot be accurately reflected by the power value displayed by the dynamometer. Many researchers have conducted relevant studies to solve this problem. A mathematical model of a vehicle transmission system was established by Irimescu et al. (2011), and a method for measuring the power output of the driving wheel and power output of the engine in a vehicle driving at uniform speed was proposed. A new method for measuring the instantaneous power output of engineering machinery wheels was proposed by Zhao et al. (2011), wherein the influence of torque imbalance on measurement accuracy was reduced by measuring the drum speed, torque, and angular acceleration. Thus, the limitation for measuring the relative balance between the drum and wheel was overcome, and the accuracy of the power measurements and other parameters were improved under unbalanced conditions. Lu et al. (2017) proposed the use of a chassis dynamometer without load sliding and back-towing no-load to measure the power loss of the chassis dynamometer. A mathematical model of the friction resistance between the drum of the chassis dynamometer and the driving wheel was established using the twice-loaded taxi method (Ouyang et al. 2015), laying a theoretical foundation for the detection value of the chassis dynamometer that more accurately reflects the actual situation of a vehicle's dynamic performance. A method for measuring the power loss of a dynamometer by reverse-towing a simulated trailer was proposed by Zhao et al. (2019), and the measurements were used to correct for the power loss due to the chassis. The results showed that the proposed approach reduced the influence of dynamometer power loss on the dynamic performance of the chassis. The non-loaded sliding method and twice-loaded taxi method were adopted by Ouyang et al. (2017), relevant data for the vehicle sliding on the chassis dynamometer were obtained, and a mathematical

model of the power output for the driving wheel was established, based on the law of conservation and transformation of energy, which established a theoretical foundation for accurate disassembly and rapid detection of automobile power output.

In the loading-sliding test experiment, the frictional resistance torque of the dynamometer and the rolling resistance between the wheel and drum change with the speed of the drum; therefore, non-loaded sliding and the twice-loaded taxi method are processes of variable acceleration, and the acceleration decreases with increases in drum speed. However, in the traditional testing process, the resistance torque is regarded as a constant, and at a certain speed interval, the variable acceleration process is considered to be a uniform speed process. In the process of measuring the rotary inertia of a dynamometer transmission system using the method of back-towing a vehicle, the measured rotational inertia includes the equivalent moment of inertia of the vehicle transmission system to the drum of the dynamometer, which reduces the precision of the mathematical model of the driving wheel power output. In the process of measuring the power output of the driving wheel, the traditional test method requires the vehicle to be in equilibrium, and the influence of the inertia moment caused by the angular acceleration of the drum on the power output test of the driving wheel is ignored.

The torque of the drum during the variable-speed running of the vehicle on the dynamometer has been analyzed, and a mathematical model of the driving wheel instantaneous power test in variable-speed driving has been established. The internal friction loss of the dynamometer and the rolling loss between the wheel and drum were considered, and the state variables of the drum speed, torque, and angular acceleration were tested using a sensor mounted on the drum connected to the shaft of the dynamometer. In the twice-loaded taxi process, the method of back-towing the trailer with the same mass as the tested vehicle was adopted, the internal frictional moment of the dynamometer and the rolling moment between the vehicle and drum were measured, and the equivalent moment of inertia of the dynamometer was obtained accurately. On this basis, a measurement method for vehicle driving wheel instantaneous power under variable-speed driving conditions is proposed, and the corresponding mathematical model is constructed, which provides technical support for the quick and accurate detection of the driving wheel power output and accurate simulation of the resistance of the driving wheel with data collected by the dynamometer.

2. Influencing factors of driving wheel power output

When a vehicle is driving on the road, the power output of the vehicle engine can be expressed using eq. 1:

$$(1) \quad P_t = P_r + P_f + P_w + P_i + P_j$$

In eq. 1, for a given vehicle, P_t represents the power output of the engine; P_r represents the power loss due to resistance in the transmission system; P_f represents the loss of power due to the rolling resistance of the driving wheel; P_w represents the loss of power due to air resistance; P_i represents the loss of power due to slope resistance; and P_j represents the loss of power from resistance to acceleration.

When a vehicle is subjected to a dynamic performance test on an indoor dynamometer test platform to simulate various types of resistance on the road, a motor is used as a load resistance device to simulate the actual driving conditions of the vehicle. When the vehicle is tested on the drum of the chassis dynamometer, the power output of the vehicle engine can be expressed by eq. 2:

$$(2) \quad P_t = P_r + P_{fc} + P_{rc} + P_s$$

In eq. 2, for a given vehicle, P_{fc} represents the loss of rolling resistance between the tires and the drum of the dynamometer; P_{rc} represents the power loss of in the transmission system from the chassis dynamometer; and P_s represents the power absorbed by the dynamometer.

Equation 2 can be transformed to eq. 3:

$$(3) \quad P_p = P_{fc} + P_{rc} + P_s$$

In eq. 3, $P_p = P_t - P_r$ for a given vehicle, where P_p represents the power output of the driving wheel. According to eq. 3, the power output of the driving wheel includes the power absorbed by the dynamometer, the power loss due to the rolling resistance between the driving wheel and the drum of the dynamometer, and the power loss due to the transmission system of the dynamometer.

In the traditional measurement method, the power output of the driving wheel is measured using a dynamometer that converts the kinetic energy of the drum into electrical energy. In our study, a torque and speed sensor was installed on the connecting shaft of the drum of the dynamometer, and the power output of the driving wheel was obtained from the relative variables of the drum. For a vehicle under constant speed: the torque balance method is used to measure the power output of the driving wheel; the torque and rotational speed of the drum are measured with a sensor; and the power output of the driving wheel is calculated from the product of torque and rotational speed. When the torque is balanced and the forces on the drum of the dynamometer are analyzed, the torque balance equation can be obtained, as shown in eq. 4:

Fig. 1. Force analysis diagram of the dynamometer drum.

$$(4) \quad T_D = T_p - T_{fc} + T_l$$

In eq. 4, for a given vehicle, T_D represents the drive torque of dynamometer drum; T_p represents the actual output torque of the driving wheel; T_{fc} represents the torque for the rolling friction between wheel and drum; T_l represents the loading resistance moment of the motor to the drum. The force analysis is shown in Fig. 1a.

Because the drum and wheel speeds of the chassis dynamometer reach equilibrium, both are in constant rotation, and no relative sliding occurs between the wheel and drum. Then, the power output of the driving wheel can be described using eq. 5:

$$(5) \quad P_p = T_p \cdot \omega = (T_{rc} + T_l + T_{fc}) \cdot \omega$$

In eq. 5, for a given vehicle, P_p represents the power output of the driving wheel; ω represents the rotational speed of the drum; and T_{rc} represents the resistance torque of the dynamometer transmission system. In Fig. 1b, α represents the angular acceleration of the drum.

When the vehicle moves at a variable speed, the torque-balance method is not applicable. When the power output of the driving wheel is variable variable-speed driving conditions, the wheel and the drum have angular acceleration so there is an imbalance between the driving torque and the resistance moment of the dynamometer drum. According to D'Alembert's principle, after adding a reverse inertial torque to the system, an equilibrium state is reached. The torque equation is given in eq. 6:

$$(6) \quad T_D - (T_{rc} + T_l) = T_p - (T_{fc} + T_{rc} + T_l) = J\alpha$$

In eq. 6, for a given vehicle, J represents the equivalent moment of inertia of the chassis dynamometer's transmission system; α represents the angular acceleration of the dynamometer drum; and $J\alpha$ represents the inertia moment. The force analysis for the chassis dynamometer drum is shown in Fig. 1b, and the power output for the driving wheel is described by eq. 7:

$$(7) \quad P_p = (T_{fc} + T_{rc} + T_l + J\alpha) \cdot \omega$$

3. Measuring the variables for the instantaneous power output of the driving wheel

3.1. Measurement of drum speed and torque

In eq. 7, for variable driving speeds, the instantaneous power output of the driving wheel is related to the resistance moment of the drum ($T_{fc} + T_{rc} + T_l$), the rotational speed of the drum (ω), the angular acceleration of the drum (α), and the equivalent moment of inertia of the chassis dynamometer's transmission system (J). Owing to the complicated structure of the chassis dynamometer, in addition to the drum and its connecting shaft, there is also a gearbox and a simulated resistance-loading motor, which leads to a large moment of inertia equivalent to the drum. Therefore, when the drum is in a non-equilibrium state, there is a large difference between the driving torque and drum resistance moment; therefore, the influence of drum angular acceleration must be considered.

The drum speed can be obtained using a magneto-electric torque sensor, the basic principle of which is shown in Fig. 2. The two ends of the elastic shaft were connected to a dynamometer and the drum. Through the elastic shaft and two groups of magneto-electric signal generators, the measured torque and speed were converted into two groups of AC signals with a phase difference.

The two sets of AC signals had the same frequency and were proportional to the shaft speed. The rotational speed pulse signal with a period of t was measured over m cycles. A standard clock pulse signal with constant frequency and period t_0 was used for counting, and the counting value N was obtained. The period t of the rotational speed pulse signal was calculated, and then the rotational speed ω was obtained from eq. 8:

$$(8) \quad \begin{cases} t = \frac{Nt_0}{m} \\ \omega = \frac{2\pi}{t} \end{cases}$$

In eq. 8, t represents the pulse period of the speed signal, and t_0 represents the standard clock pulse signal cycle.

When torque was applied at the sensor, elastic deformation was produced by the elastic shaft. The phase difference between the two groups of AC signals was proportional to the torque on the magneto-elastic axis. When the elastic shaft was not subject to torque, the initial phase angle was π . When the elastic shaft was subject to torque, the two gears rotated relative to each other, and the change in the relative rotation angle for each tooth was ϕ . The change in rotation angle was caused by the torque, which is the sum of the friction resistance moment between the wheel and the drum (T_{fc}), the resistance moment of the dynamometer driving system (T_{rc}), and the loading-resistance moment of the dynamometer loading motor to the drum (T_l), such that,

Fig. 2. Diagram showing the working principles of the sensor.

$T_{fc} + T_{rc} + T_l = T_s$, where T_s is the torque displayed by the sensor. Overall, ϕ can be expressed using eq. 9:

$$(9) \quad \phi = \frac{(T_{fc} + T_{rc} + T_l) \cdot L}{kGJ_s} = \frac{T_s L}{kGJ_s}$$

In eq. 9, ϕ represents the phase difference for the induced electromotive force generated by two coils; L represents the working length of the sensor's elastic axis; G represents the elastic modulus of the elastic axis; J_s represents the moment of inertia of the elastic axis; and k represents the number of teeth in the signal gear. These parameters are related to the structural properties of the sensor itself.

3.2. Measurement method for angular acceleration of drum

Because the number of sensor-signal gear teeth (k) is known, the angle between each of the adjacent teeth is $\theta = 2\pi/k$, and the sensor signal of the gear teeth is known. If the time taken by the gear to rotate from the i th tooth to the $i+1$ tooth is denoted as Δt_i , then the average rotational speed of the drum is $\omega = \theta/\Delta t_i$ in the period of time Δt_i . If the time taken by the gear to rotate from the $i+1$ tooth to the $i+2$ tooth is denoted as Δt_{i+1} , then the average rotational speed of the drum is $\omega_i = \theta/\Delta t_{i+1}$ in the period of time Δt_{i+1} . The average angular acceleration of the drum ($\bar{\alpha}$) during this period is expressed by eq. 10. Because Δt_i is very short, $\bar{\alpha}$ can be taken as the acceleration α_i :

$$(10) \quad \bar{\alpha} = \frac{2(\omega_{i+1} - \omega_i)}{\Delta t_i + \Delta t_{i+1}} = \frac{2\theta(\Delta t_i - \Delta t_{i+1})}{\Delta t_{i+1}\Delta t_i(\Delta t_i + \Delta t_{i+1})}$$

3.3. Measuring the instantaneous power output for the driving wheel

When the dynamometer is running, an accurate value for rotational inertia of the transmission system of the chassis dynamometer is key to accurately simulating the driving resistance of a vehicle. The traditional twice-loaded method is based on the chassis dynamometer for two different load torques, following the law of conservation of energy.

The traditional testing process includes the following four steps:

1. The resistance moment of the anti-towing experimental vehicle at the speed of v_0 was measured using the non-loaded sliding method. The experimental vehicle was accelerated to speed v_1 on the chassis dynamometer; $v_1 > v_0$ (the kinetic energy of the experimental vehicle) was stored by the dynamometer, and the vehicle glided under the joint action of tire rolling resistance and dynamometer transmission system resistance after cutting off the vehicle's power. When the experimental vehicle slid to speed v_2 , $v_2 < v_0$, taxi distance s_{v_0} was recorded, $v_0 = (v_1 + v_2)/2$, the mathematical model of the sliding process can be expressed by eq. 11:

$$(11) \quad \begin{cases} T_{v_0} \cdot \theta = \frac{1}{2} J_1 \frac{(v_1^2 - v_2^2)}{r^2} \\ \theta = \frac{s_{v_0}}{r} \end{cases}$$

In eq. 11, T_{v_0} represents the sum of T_r , T_{fc} , and T_{rc} when the vehicle is driving at the speed of v_0 ; s_{v_0} represents the distance of the vehicle sliding on the dynamometer drum from the speed v_1 to v_2 ; r represents the radius of the dynamometer drum; θ represents the angle of the vehicle sliding on the dynamometer drum from speed v_1 to v_2 ; J_1 represents the equivalent moment of inertia to the drum of the vehicle's transmission system and chassis dynamometer.

2. The experimental vehicle was run on the dynamometer drum at speed v_1 , the drum's rotational speed was ω_1 , and then the vehicle's power system was cut off. At the same time, the experimental vehicle was loaded with a resistance moment (T_1) using the dynamometer. The vehicle slid under the combined action of the loading-resistance moment (T_1), frictional resistance moment of the vehicle transmission system (T_r), rolling friction resistance moment between the drum and the wheel (T_{fc}), and resistance moment of the dynamometer transmission system (T_{rc}). The vehicle taxied to speed v_2 , the drum speed was ω_2 , and the distance was S_1 after the first slide had been completed.
3. In the second slide process, the experimental vehicle was loaded with a different load resistance moment (T_2) using the dynamometer. The initial and final speeds of v_1 and v_2 in the sliding process were kept the same as the first loading-sliding process. The taxi distance (S_2) was recorded.
4. Because the speeds v_1 , v_2 , and v_0 were almost identical, the frictional resistances produced by the chassis dynamometer at the three speeds were approximately equal, according to the mathematical model produced with a kinetic energy theorem (eq. 12):

$$(12) \quad \begin{cases} (T_1 + T_{v_0}) \cdot \theta_1 = \frac{1}{2} J_1 \frac{(v_1^2 - v_2^2)}{r^2} = \frac{1}{2} J_1 (\omega_1^2 - \omega_2^2) \\ (T_2 + T_{v_0}) \cdot \theta_2 = \frac{1}{2} J_1 \frac{(v_1^2 - v_2^2)}{r^2} = \frac{1}{2} J_1 (\omega_1^2 - \omega_2^2) \\ \theta_1 = \frac{s_1}{r} \\ \theta_2 = \frac{s_2}{r} \end{cases}$$

In eq. 12, T_1 represents the load resistance moment of the dynamometer to the drum during the first loaded slide; T_2 represents the load resistance moment of the dynamometer to the drum during the second loaded slide; θ_1 represents the rotation angle of the vehicle sliding from v_1 to v_2 on the chassis dynamometer drum when the loaded torque was T_1 ; θ_2 represents the rotation angle of the vehicle sliding from v_1 to v_2 on the chassis dynamometer drum when the loaded torque was T_2 . T_{v_0} and J_1 are described by eq. 13:

$$(13) \quad \begin{cases} J_1 = \frac{2(T_2 - T_1)\theta_1\theta_2}{(\theta_1 - \theta_2)(\omega_1^2 - \omega_2^2)} \\ T_{v_0} = \frac{(T_2 - T_1)\theta_1\theta_2}{(\theta_1 - \theta_2)\theta} \end{cases}$$

In the test process of the traditional twice-loaded taxi method, because the anti-towing sliding is performed with an actual vehicle, the resistance moment of the driving system in the experimental vehicle is described by T_{v_0} . Based on the empirical formula in eq. 13, 15% of the engine power output was set as the power loss of the vehicle transmission system. We produced a mathematical model using the variables measured by the twice-loaded taxi method, a mathematical model of the power output from the driving wheel, which is described in eq. 14:

$$(14) \quad P_p = (1 - \alpha) \left[\frac{\theta_1\theta_2(T_2 - T_1)v_0}{\theta(\theta_1 - \theta_2)r} + P_s \right]$$

In eq. 14, $\alpha = 15\%$, and $(1 - \alpha)$ represents the efficiency of the chassis transmission system.

According to this model, the output torque of the driving wheel (T_p) at speed is calculated using eq. 15:

$$(15) \quad T_p = \frac{(1 - \alpha) \left[\frac{\theta_1\theta_2(T_2 - T_1)v_0}{\theta(\theta_1 - \theta_2)r} + P_s \right] r}{v_0}$$

In the test process of the traditional twice-loaded taxi method, there are three main drawbacks that cause the mathematical model and measurement results to be inaccurate. First, owing to the back-towing of the experimental vehicle, some parts of the vehicle transmission system (including the main reducer, differential, half axle, and driving wheel) move together in the dynamometer's anti-towing vehicle. Therefore, because the connection between the driving wheel and the vehicle's transmission system cannot be cut-off

effectively, the equivalent moment of inertia of the dynamometer obtained by measurement includes the equivalent moment of inertia of the vehicle's transmission system. The measured moment of inertia includes the equivalent moment of the vehicle transmission system to the drum, not the chassis dynamometer system, and the influence of the power loss of the vehicle transmission system on the measurement results cannot be completely excluded. The accuracy of the mathematical model of the instantaneous power output of the vehicle driving wheel is significantly affected by inaccurate measurement results. Second, when three different speeds are selected for the operating conditions, the friction resistance generated by the dynamometer is regarded as approximately the same as that in the traditional twice-loaded taxi method. Third, the traditional method only considers the torque of the drum under the state of equilibrium without considering the inertia moment of the drum, but the actual loading and sliding process is a variable-speed process.

To eliminate the influence of the vehicle's transmission system on the measurement of the equivalent moment of inertia of the dynamometer transmission system, a twice-loaded taxi measurement method for a back-towed simulated trailer is proposed.

Improved testing methods:

1. A universal trailer with an adjustable wheelbase, as shown in Fig. 3, was constructed to simulate different parameters with different applied weights. The center of gravity of a vehicle could be adjusted by moving the weight. The purpose of the trailer was to measure the dynamometer's equivalent moment of inertia more accurately, and the influence of the vehicle's transmission system on the measurements of the power output of the driving-wheel was avoided.
2. A chassis dynamometer was used to drag the trailer to reach speed v ($v > v_0$). The kinetic energy of the trailer was stored by the chassis dynamometer, and then a drag torque T_1 was exerted on the experimental trailer. The trailer slid under the combined action of the resistance moment (T_1), the rolling friction resistance moment (T_{fc}) between the drum and the wheel, and the resistance moment (T_{rc}) of

Fig. 3. Universal trailer with adjustable wheelbase.

the dynamometer transmission system. The rotational speed of the dynamometer drum and the angular acceleration at the moment of rotational speed $\omega_0 = v_0/r$ were measured.

3. In the second sliding process, the experimental vehicle was loaded with a resistance moment, T_2 , which was different from T_1 , and the rotational speed of the dynamometer drum ($\omega_0 = v_0/r$) and the angular acceleration (α_2) at the moment of rotational speed were collected. A mathematical model was constructed using the angular momentum theorem as described by eq. 16:

$$(16) \quad \begin{cases} T_1 + T_{fc} + T_{rc} = J\alpha_1 \\ T_2 + T_{fc} + T_{rc} = J\alpha_2 \end{cases}$$

In eq. 16, α_1 represents the angular acceleration of the drum at angular velocity ω_0 during the first loaded slide; α_2 represents the angular acceleration of the drum at angular velocity ω_0 during the second loaded slide. The equivalent moment of inertia of the dynamometer's transmission system J is shown in eq. 17, and $T_{fc} + T_{rc}$ at angular velocity ω_0 is described in eq. 18:

$$(17) \quad J = \frac{T_1 - T_2}{\alpha_1 - \alpha_2}$$

$$(18) \quad T_{fc} + T_{rc} = \frac{(T_1 - T_2)\alpha_1}{\alpha_1 - \alpha_2} - T$$

4. The instantaneous power output (P_p) and torque (T_p) of the driving wheel are described in eq. 19:

$$(19) \quad \begin{cases} P_p = \left(T_{fc} + T_{rc} + T_1 + \frac{T_1 - T_2}{\alpha_1 - \alpha_2} \alpha \right) \cdot \omega = \left(T_s + \frac{T_1 - T_2}{\alpha_1 - \alpha_2} \alpha \right) \cdot \omega \\ T_p = T_s + \frac{T_1 - T_2}{\alpha_1 - \alpha_2} \alpha \end{cases}$$

In eq. 19, T_s represents the torque as indicated by the sensor, which is the sum of the load resistance moment of dynamometer (T_1), the rolling friction resistance moment between the drum and wheel (T_{fc}), and the resistance moment of the dynamometer transmission system (T_{rc}).

Fig. 4. Structure of the chassis dynamometer system. **Fig. 5.** Signal acquisition and processing system.

4. Construction of the driving wheel instantaneous power output measuring system

4.1. Construction of the measuring system

A single-drum chassis dynamometer was used as the test platform, and an asynchronous electric dynamometer was used as the vehicle's road-resistance-loading device, which integrates the vehicle chassis power output and anti-towing measurements. Once the rotational speed and torque sensors are placed on the connecting shaft of the dynamometer drum and connected to the transmission system of the dynamometer, the rotational speed, torque, and angular acceleration of the dynamometer drum can be directly measured by the sensor. The measured signal was processed and transmitted to the dynamometer control system, and the instantaneous power output torque of the driving wheel was obtained using the mathematical model proposed in this study. Meanwhile, the output torque was loaded onto the motor as the simulated resistance moment of the vehicle driving on a road. Figure 4 is a diagram of the system.

An STM32 (a single-chip microcomputer) was the core of the signal acquisition and processing system. Two voltage signals approximating a sine wave were sent out by the signal gear in the rotation-torque sensor. After filtering, amplifying, and shaping, the two channels of the signals were converted into two channels of square-wave signals with voltage signals of the same

frequency approximating a sine wave and a void ratio of 50%. One of the square-wave signals was transmitted to the STM32 as the input signal for the speed measurements. The phase difference pulse signal was obtained from a bi-stable trigger (74LS74) and transmitted to the STM32 for drum torque measurements. Following eq. 7, to measure angular acceleration, Δt_i was measured continuously, the counter was set to rise along the trigger count, and a single chip microcomputer was used to capture and record the rising edge of the square wave signal to obtain Δt_i , which was constantly measured and recorded for continuous measurement of the square wave signal pulse width to create conditions. The average angular acceleration in two adjacent periods was calculated using the square-wave pulse width at any two adjacent moments, and eq. 8. Figure 5 shows the signal acquisition and processing system is. The speed, torque, and angular acceleration signals acquired by the STM32 were transferred to an industrial control computer using a dual driver/receiver (MAX232).

4.2. Real-time performance and robustness of the measurement method and system

4.2.1. Analysis of real-time performance

The real-time performance of the measurement algorithm for the power output of the driving-wheel was mainly evaluated based on the time consumed in the calculation process and the process of measuring the related variables:

1. Real-time performance of the calculation process for the instantaneous power output of the driving wheel

Using the method proposed in this study, the instantaneous power output of the driving wheel can be obtained using eq. 19. Before this equation can be used, it is necessary to measure four physical quantities of the vehicle while running on the drum of the dynamometer, namely, the equivalent moment of inertia of the dynamometer transmission system (J), the rotational speed (ω), the torque displayed by the sensor (T_s), and the angular acceleration of the drum (α). The mathematical model and the calculation process are simple, because only a few steps using simple arithmetic operations need to be completed, so the calculation process consumes less time than the traditional method. An STM32 series single-chip microcomputer was used as the core of the measurement system. This type of processor can perform a single calculation in less than 10 μ s to satisfy the real-time requirements of the algorithm and measurement system.

2. Analysis of the real-time measurements of variables related to the power output of a driving wheel

As described by eq. 19, the calculations for the instantaneous power output of the driving wheel involve four physical quantities: J , ω , T_s , and α . The real-time analysis of the measurement process for the four physical quantities is as follows.

- (a) The equivalent moment of inertia of the chassis dynamometer transmission system (J ; a constant) was obtained with an experimental twice-loaded taxi method incorporating a back-towed simulated trailer.
- (b) The rotational speed of drum (ω) was measured using the period method, and the mathematical model is described by eq. 8. When the vehicle was driven at low speed on the dynamometer, the drum speed of the dynamometer was approximately 10 rad/s and the period was approximately 0.628 s. With 60 teeth on the signal gear in the magnetoelectric transducer, the maximum signal pulse period t generated by the signal gear was approximately 10 ms, obtained from the number of crystal oscillator pulses in the pulse signal cycle. Therefore, the rotational speed measurement was completed within a maximum of 10 ms, which satisfies the requirement for real-time measurements.
- (c) T_s was obtained by measuring the phase difference between the two speed signals. The mathematical model is given in eq. 9. The frequency of the phase difference signal was the same as the signal period t produced by the signal gear, the maximum pulse period of the phase difference signal was

approximately 10 ms, and the phase difference (ϕ) was obtained using the period method to record the number of crystal pulses in the period of the pulse width.

- (d) The angular acceleration (α) was obtained using a mathematical model (eq. 10). In essence, it was obtained by calculating the period (t) of adjacent pulse signals in the signal generated by the measurement signal gear, and calculation of the angular acceleration was simple. Essentially, it was calculated by measuring the amount of time (Δt_i and Δt_{i+1}) of two adjacent cycles of the speed signal. The calculation process for angular acceleration is simple.

Therefore, the measurement methods for the three physical quantities measured using the sensor (T_s , ω , and α) are simple and are related to the signal period (t) generated by the sensor signal gear, which is approximately 10 ms at its maximum. In essence, the measurement process is a problem when measuring the pulse width using a periodic method. The calculation processes for all types of measured variables and instantaneous power output are simple arithmetic operations that do not require iteration operations or other time-consuming processes. Therefore, whether the signal generated by the sensor is processed and calculated to obtain the measured variable or the instantaneous power output of the vehicle driving wheel is calculated using the established mathematical model, the maximum calculation period of the power output is approximately 10 ms. With an increase in driving speed, the speed signal cycle decreases, the signal measurement time decreases, and the real-time performance improves. Because the STM32 microcontroller is used as the core of the signal acquisition and processing system, the requirement for measuring the instantaneous power of the driving wheel can be satisfied.

4.2.2. Analysis of robustness

We used the mathematical model of angular acceleration (α ; eq. 10) to determine the robustness of the algorithm under high acceleration, because vehicle acceleration is obtained by measuring the angular acceleration of the dynamometer drum. According to the mathematical model, α can be obtained by calculating the period (t) of the adjacent pulse signals in the signal generated by the measurement signal gear. The calculation of α is reliable, and the accuracy of the measurement model and stability of the algorithm do not depend on the angular acceleration. Simultaneously, after amplifying, filtering, and shaping the approximately sinusoidal AC electrical signal generated by the sensor, the signal conditioning circuit effectively collects real-time data. Moreover, the noise can be effectively filtered out by the filter circuit, and pulse spike

Table 1. Modified twice-loaded sliding experimental data and the equivalent moment of inertia for the chassis dynamometer as obtained from the data.

v (km/h)	ω (rad/s)	$\bar{\alpha}_1$ (rad/s ²)	$\bar{\alpha}_2$ (rad/s ²)	J (kg $\times$ m ²)
100	45.54	0.340	0.230	454.5
80	36.43	0.316	0.207	458.7
60	27.32	0.295	0.187	463.0
40	18.22	0.277	0.168	458.6

Table 2. Test variables obtained at different speeds and the instantaneous power output and output driving torque for the driving wheel, as calculated using our mathematical model.

v (km/h)	ω (rad/s)	T_S (N $\times$ m)	α (rad/s ²)	T_P (N $\times$ m)	P_P (W)
40	18.22	32.2	0.157	104.2	1899
45	20.50	49.1	0.151	118.4	2427
50	22.77	66.6	0.146	133.6	3042
55	25.05	85.8	0.141	150.5	3770
60	27.32	106.8	0.134	168.3	4598
65	29.60	127.8	0.128	186.5	5520
70	31.88	149.3	0.122	205.3	6545
75	34.15	173.3	0.115	226.1	7721
80	36.43	198.2	0.109	248.2	9034
85	38.70	231.6	0.100	277.5	10739
90	40.98	262.1	0.092	304.2	12472
95	43.26	297.6	0.087	337.5	14600
100	45.54	324.5	0.081	361.6	16467
105	47.82	363.5	0.063	392.2	18765
110	50.10	395.1	0.043	414.8	20783

interference is avoided. A stable square wave with the same frequency as the AC electric signal can be used to measure angular acceleration (α) under different acceleration conditions.

5. Analysis and verification of experimental results

The power output from the driving wheel of the 2009 Jiangling Canon (Jiangling Motors Co., Ltd., China) was tested using a single-drum chassis dynamometer (48 mm AVL ROADSIM™; AVL List GmbH, Austria) with a drum radius (r) of 0.61 m. Using the measurement method proposed in this paper, reverse towing and load taxiing were carried out on the trailer of the simulated 2009 Jiangling Canon. The values for loading-resistance moment were set as $T_1 = 100$ N $\times$ m and $T_2 = 50$ N $\times$ m, and the test speed was set to $v_0 = 100, 80, 60$, and 40 km/h. The angular acceleration of the drum was set to two loading-resistance values (α_1 and α_2), which were tested five times; the average angular acceleration values are denoted as $\bar{\alpha}_1$ and $\bar{\alpha}_2$, and the moment of inertia of the chassis dynamometer transmission system was calculated using eq. 17. The results are presented in Table 1.

The equivalent moment of inertia of the drum chassis dynamometer transmission system is calculated using

data collected from a sliding experiment and the mathematical model proposed in this paper, and the average of the moment of inertia (J) was 458.7 kg $\times$ m². According to official data provided by the Jiangling Motor Co. (Ouyang et al. 2016), the equivalent mass of the 48 mm AVL ROADSIM dynamometer was 1178 kg. Because of the simulated trailer sliding during the experiment, the moment of inertia obtained, which was calculated as $J = 454.2$ kg $\times$ m², included the equivalent moment of inertia of the wheel to the drum. The difference between the calculated results and the official data was 1%. Because of the influence from tire pressure and deformation during driving, the equivalent moment of inertia calculated with the experimental data from the twice-loaded taxiing was used in the mathematical model of the power output for the driving wheel.

The instantaneous power output of the driving wheel during vehicle acceleration was tested at 15 different driving speeds within the range of 40–110 km/h. The values for ω , T_S , and α were measured by the sensor mounted on the drum's connecting shaft, and the instantaneous power output (P_P) and output driving torque (T_P) of the driving wheel were calculated using eq. 19. The measured data and values calculated using the mathematical model are presented in Table 2.

Table 3. Data comparison for the power output of the driving wheel and torque obtained using different models.

v (km/h)	ω (rad/s)	Traditional twice-loaded taxi method		Proposed method		Vehicle road sliding experiment	
		T'_p (N × m)	P'_p (W)	T_p (N × m)	P_p (W)	T_{road} (N × m)	P_{road} (W)
40	18.22	102.2	1862	104.2	1899	103.7	1889
45	20.50	116.2	2388	118.4	2427	117.7	2413
50	22.77	131.2	2997	133.6	3042	133.0	3028
55	25.05	155.6	3742	150.5	3770	152.1	3810
60	27.32	164.4	4492	168.3	4598	167.2	4567
65	29.60	183.3	5426	186.5	5520	185.1	5479
70	31.88	208.3	6643	205.3	6545	206.1	6570
75	34.15	230.9	7884	226.1	7721	226.6	7740
80	36.43	253.7	9244	248.2	9034	248.5	9047
85	38.70	284.8	11 021	277.5	10 739	278.1	10 762
90	40.98	298.8	12 244	304.2	12 472	306.1	12 550
95	43.26	324.4	14 035	337.5	14 600	336.2	14 561
100	45.54	347.1	15 853	361.6	16 467	363.5	16 556
105	47.82	377.0	18 026	392.2	18 765	396.3	18 946
110	50.10	396.5	19 856	414.8	20 783	418.8	20 981

To compare and analyze the experimental data, the output torque model for the driving wheel was measured, and the results from the road driving experiments were compared and analyzed with the instantaneous output torque of the driving wheel obtained by the method used in this study. The experimental road driving data were provided by the Jiangling Motor Co. The power output of the driving wheel was calculated from the road-driving resistance multiplied by the speed.

In Table 3, the output torque of the driving wheel and power output of the driving wheel, as determined using the mathematical model established using the traditional twice-loaded taxi method, are denoted as T'_p and P'_p , respectively. The vehicle's road-resistance moment and power output of the driving wheel obtained through the road sliding experiments are denoted as T_{road} and P_{road} , respectively. The output torques of the driving wheel, as determined using the two models and the road-resistance moment at different speeds, are shown in Fig. 6.

According to the data in Fig. 6, with an increase in vehicular speed within the range of 40–110 km/h, the output torque of the driving wheel gradually increases, and the power output of the driving wheel also gradually increases. By using the experimental method and mathematical model proposed in this paper, the output torque of the driving wheel is closer to the road driving resistance, and the road driving resistance can be more effectively simulated. The driving wheel power output was closer to the road-driving conditions, and the test result was more accurate.

To further analyze the accuracy of the model, the output torques of the driving wheel, as obtained using the traditional method and the method in this study,

Fig. 6. The output torque for the driving wheel at different speeds. **Fig. 7.** The relative errors for the power output of the driving wheel as obtained using the mathematical model and the traditional method, compared with road driving resistance.

were compared with the road driving resistance moment, and the relative errors of the power output of the driving wheel for the two models relative to the road-driving model were obtained, as shown in Fig. 7.

Table 4. Relative error of the instantaneous power output.

$\frac{ P'_p - P_{road} }{P_{road}} \times 100\%$		$\frac{ P_p - P_{road} }{P_{road}} \times 100\%$	
Max. relative error	Mean relative error	Max. relative error	Mean relative error
5.32	2.48	1.06	0.57

The model from Lu et al. (2017) and the model proposed in this paper were used to calculate the instantaneous power output of the driving wheel of the vehicle, within the driving speed range of 40–110 km/h (Fig. 7), and then compared with the instantaneous power output of the driving wheel for road-driving provided by the Jiangling Motor Co. The mean and maximum relative errors are listed in Table 4.

Using eq. 9, the maximum relative error was 5.32%, and the mean relative error was 2.48%. As shown in Fig. 7, the relative error was analyzed across the entire acceleration process, and the measurement error gradually increased with increased speed of the vehicle. When the speed was 110 km/h, the maximum phase error was 5.32%. The power output of the driving wheel was compared with the road-running data using the mathematical model proposed in this paper. The maximum relative error was 1.06%, and the mean relative error was 0.57%, which was far less than those from the measurement model and method (eq. 9), and the relative error was not affected by the vehicle's speed. Therefore, the mathematical model of the power output for the driving wheel described in this paper more accurately reflects road driving, and the relative error is not affected by the vehicle's speed.

6. Conclusions

To solve the problem of insufficient precision for the measurements of instantaneous power output from the driving wheel, as determined using the chassis dynamometer in the traditional test method, a measurement system was constructed incorporating the drum speed, torque, angular acceleration, and other variables of the dynamometer. On this basis, a model of the instantaneous power output for the driving wheel was established, and a new method for measuring the instantaneous power output of the driving wheel is proposed as follows.

1. We propose a test method involving anti-towing and sliding of the trailer in the twice-loaded taxiing method. The experimental data for the drum's rotational speed, angular acceleration, torque, and other variables related to the motion of the drum are directly measured with a sensor directly, and the equivalent moment of inertia of the chassis dynamometer is calculated. With this method, we avoid the influence of the vehicle chassis

transmission system on the measurement results (as in the traditional test method), and the errors introduced by using an empirical formula to calculate the power output of the driving wheel. The error between the measured results for the equivalent rotation of the dynamometer obtained by this method and the official data was only 1%, and accurate parameter values for measuring the instantaneous output torque of the driving wheel are provided.

2. The measurement and calculation of the instantaneous power output of the driving wheel includes values for J , α , ω , and T_s , as well as the calculated power output of the driving wheel. The measurement process uses the period method to measure the speed signal period generated by the sensor, and the calculation process uses a mathematical model. By analyzing the duration of the calculation process for the power output of the driving wheel and the measurement of the relevant variables in the model, we observed that at low speeds, the signal cycle is large and the measurement time is long. With increased speed, the signal cycle was shortened. The models of the measured variables and instantaneous power output are all constant and do not change with time and speed. The calculation process is a simple arithmetic operation, and there are no time-consuming processes, such as iterative operations. An STM32 microcontroller was used as the core of the signal acquisition and processing system so that the instantaneous power of the driving wheel could be measured in real time.

The values for angular acceleration (α), using the model, are constant; the measurement results only depend on the adjacent cycles of the speed signal, so the calculation process is reliable, there is no oscillation, chaos, divergence, or other unexpected cases, the essence of the algorithm is non-dynamic, and the measurement method is robust under high acceleration.

3. We established a mathematical model of the instantaneous power output of a driving wheel, taking into consideration the force of the driving wheel on the drum of the dynamometer and in accordance with the law of conservation of angular momentum. Compared with the traditional model, all of the original data in the new model were

measured directly by the sensor. The shortcoming of the traditional method in adopting the internal resistance moment of the dynamometer as a constant force, and thus introducing error, is overcome by our method, and the calculation process is more accurate and simple. When we tested the power output of the driving wheel at 40–110 km/h, the maximum relative error was 1.06% and the mean relative error was 0.57%. Compared with the measurement error of the traditional model (the maximum relative error was 5.32%, and the average relative error was 2.48%), the accuracy of the measurement is significantly improved, and the output torque of the driving wheel obtained by the model is closer to the output torque of the driving wheel as measured during the road experiments, indicating that the dynamometer can accurately simulate the road resistance of the vehicle.

4. The power output of the driving wheel estimated using our mathematical model was not affected by the speed of the vehicle, which overcomes the shortcomings of the traditional model, in which the calculation error increases with increased speed of the vehicle. The power value obtained using the new model more accurately reflects the instantaneous power value of the driving wheel. The precision of the measurements was significantly improved compared with the traditional method, and a theoretical foundation for the accurate no-demolition rapid detection of the power output of the vehicle is provided. To further improve the accuracy of the model of the instantaneous power output of the driving wheel and meet the requirements of various types of vehicle tires in driving environments with different road conditions, temperature, and humidity, the viscous effect will be considered by adding the adhesive force between the wheel and drum to the calculation model as a source of resistance, in a future study.

Acknowledgments

We thank China Postdoctoral Science Foundation (Grant No. 2015M572729) for providing the funding for this study.

References

Chelia, F., Braghina, F., Brusaroscob, M., Mancosub, F., and Sabbionia, E. 2011. Design and testing of an innovative

measurement device for tyre–road contact forces. *Mech. Syst. Sig. Process.* **25**(6): 1956–1972. doi:[10.1016/j.ymssp.2011.02.021](https://doi.org/10.1016/j.ymssp.2011.02.021).

- Chen, L., Wang, Z., Liu, S., and Qu, L. 2018. Using a chassis dynamometer to determine the influencing factors for the emissions of Euro VI vehicles. *Transportation Research Part D: Transport and Environment*. **65**(12): 564–573. doi:[10.1016/j.trd.2018.09.022](https://doi.org/10.1016/j.trd.2018.09.022).
- Irimescu, A., Mihon, L., and Pădure, G. 2011. Automotive transmission efficiency measurement using a chassis dynamometer. *Int. J. Automot. Technol.* **12**(4): 555–559. doi:[10.1007/s12239-011-0065-1](https://doi.org/10.1007/s12239-011-0065-1).
- Lu, J., Ouyang, A., Liu, Y., and Wang, J. 2017. Mathematical model about indoor precision measurement to the output power of vehicle driving wheel. *Machine Des. Res.* **33**(8): 146–149. doi:[10.1006/2343\(2017\)06-146-04](https://doi.org/10.1006/2343(2017)06-146-04).
- Nakashima, Y., and Kajii, Y. 2017. Determination of nitrous acid emission factors from a gasoline vehicle using a chassis dynamometer combined with incoherent broadband cavity-enhanced absorption spectroscopy. *Sci. Total Environ.* **575**(1): 287–293. doi:[10.1016/j.scitotenv.2016.10.050](https://doi.org/10.1016/j.scitotenv.2016.10.050).
- Ouyang, A., Xiao, W., and Chen, Q. 2015. Accuracy of single drum chassis dynamometer for measuring the rolling resistance of tires. *China Measurement Test*. **41**(8): 13–19. doi:[10.11857/j.issn.1674-5124.2015.08.003](https://doi.org/10.11857/j.issn.1674-5124.2015.08.003).
- Ouyang, A., Bi, P., Liu, Y., and Liu, Z. 2016. Study on main parameter determination of single cylinder chassis dynamometers. *China Measurement Test*. **42**(3): 7–11. doi:[10.118-57/j.issn.1674-5124.2016.03.002](https://doi.org/10.118-57/j.issn.1674-5124.2016.03.002).
- Ouyang, A., Lu, J., Liu, Y., and Huang, J. 2017. Research on the measurement model of the frictional resistance between the roller of chassis dynamometers and driving wheels. *Modular Machine Tool Auto. Manufact. Technique*. **8**(8): 1–5. doi:[10.13462/j.Cnki.mmtamt.2017.08.001](https://doi.org/10.13462/j.Cnki.mmtamt.2017.08.001).
- Páscoa1, J.C., Brójo, F.P., Santos, F.C., and Fael, P.O. 2012. An innovative experimental on-road testing method and its demonstration on a prototype vehicle. *J. Mechan. Sci. Technol.* **26**(6): 1663–1670. doi:[10.1007/s12206-012-0413-8](https://doi.org/10.1007/s12206-012-0413-8).
- Sun, Y., Wang, Y., Zhu, R., Geng, R., Zhang, J., Fan, D., et al. 2018. Development of test bed of hybrid electricvehicle based on chassis dynamometer. *IOP Conf. Series: Mater. Sci. Eng.* **452**(4): 1–7. doi:[10.1088/1757-899x/452/4/042112](https://doi.org/10.1088/1757-899x/452/4/042112).
- Triantafyllou, G., Dimaratos, A., Ntziachristos, L., Bernard, Y., Dornoff, J., and Samarasa, Z. 2019. A study on the CO₂ and NO_x emissions performance of Euro 6 diesel vehicles under various chassis dynamometer and on-road conditions including latest regulatory provisions. *Sci. Total Environ.* **666**(20): 337–346. doi:[10.1016/j.scitotenv.2019.02.144](https://doi.org/10.1016/j.scitotenv.2019.02.144).
- Zhao, W., Wang, Q., Jing, Q., and Wang, L. 2011. Research on measurement method of wheel instantaneous power based on AT89S52. *Instru. Tech. Sensor*. **31**(8): 73–76. doi:[10.1080/0144929X.2011.553739](https://doi.org/10.1080/0144929X.2011.553739).
- Zhao, W., Wang, Q., and He, X. 2019. Error analysis and method research of construction machinery chassis dynamic performance experiment based on dynamometer. *China Measurement Test*, **45**(5): 151–156. doi:[10.11857/j.issn.1674-5124.2017120104](https://doi.org/10.11857/j.issn.1674-5124.2017120104).