ACCOUNTING FOR COVARIANCES AMONG TEST DAY MILK YIELDS IN DAIRY COWS

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Daily milk weights from 1006 lactations on 775 Holstein-Friesian cows in 42 herds and monthly test-day weights from 102 540 lactations on 73 717 cows in 17 481 herd-year-seasons were used to study the influence of covariances among milk weighings within a lactation on three models for describing the shape of the lactation curve for individual cows. The models included a gamma function, an inverse quadratic polynomial function, and a regression model of yields on day in lactation (linear and quadratic) and on log of 305 divided by day in lactation (linear and quadratic). For each model, several variance-covariance matrices of the observation vector were used. Models were compared on the basis of squared deviations of predicted versus actual milk weights and on the correlation between predicted and actual weights through the lactation averaged over cows. Better predictions were observed when covariances among test-day yields were ignored while models could be ranked regression model, gamma function, and inverse quadratic polynomial function in order of best to worst. Heritability estimates for the parameters of the various models and for 305-d milk yield ranged from 0.11 to 0.30. Genetic correlations were estimated and predictions of correlated responses in 305-d yield from selecting on various combinations of parameters from each method were computed. The best combination of parameters of the gamma function gave a relative efficiency of 74.7% as compared to selection for 305-d yield alone.

Key words: Lactation curves, covariances, Holsteins

[Prise en compte de la covariance dans l'étude du rendement laitier des vaches laitières au jour d'épreuve.]
Titre abrégé: Covariance et rendements quotidiens.
Nous avons utilisé le poids quotidien du lait produit en 1006 lactations par 775 vaches Holstein-Friesian appartenant à 42 troupeaux et le poids mensuel, mesuré au jour d'épreuve, correspondant à 102 540 lactations de 73 717 vaches dans 17 481 troupeaux-années-saisons pour déterminer les effets de la covariance entre les valeurs du poids du lait pour une lactation donnée sur trois modèles permettant de décrire la forme de la courbe de lactation de vaches individuelles. Les modèles comprenaient une fonction gamma, une fonction polynomiale quadratique inverse et un modèle de régression des rendements sur le nombre de jours en lactation (linéaire et quadratique) et sur le logarithme de 305 divisé par le nombre de jour en lactation (linéaire et quadratique). Pour chaque modèle, nous avons utilisé plusieurs matrices de variance-covariance du vecteur observations. La comparaison des modèles était fondée sur la valeur du carré des écarts des poids de lait prévus sur les poids de lait réels ainsi que sur la corrélation entre les poids prévus et les poids réels pendant une période de lactation moyenne. Les prévisions se sont avérées plus exactes lorsqu'on ignorait les covariances entre les rendements au jour d'épreuve et les modèles pouvaient être classés comme suit, en ordre décroissant d'exactitude des résultats obtenus: modèle de régression, fonction gamma et fonction polynomiale quadratique inverse. Les valeurs estimatives de

l'héritabilité pour les paramètres des divers modèles et pour le rendement laitier en 305 jours variaient de 0,11 à 0,30. Nous avons évalué les corrélations génétiques et calculé les prévisions des réactions mises en corrélation du rendement de 305 jours en choisissant diverses combinaisons de paramètres de chaque méthode. La meilleure combinaison des paramètres de la fonction gamma a donné une efficacité relative de 74,7%, comparativement à la sélection fondée uniquement sur le rendement de 305 jours.

Mots clés: Courbes de lactation, covariance, Holsteins

Up to 12 supervised weighings of daily milk yield per lactation are usually taken on cows in herds enrolled on official milk recording programs. The test-day yields are used to predict total 305-d yields which are used to evaluate the additive genetic merit of sires and cows. Most methods of predicting 305-d yields do not consider the covariances among test-day yields. Covariances occur between adjacent test-day yields due to common environmental factors such as weather and feed, and due to repeatability of yields on the same cow. Utilization of the covariances should improve the accuracy of 305-d predictions.

A graph of daily yields against days since calving can be separated into three parts: the increasing slope from calving to about 35 d, the peak, and the decreasing slope from the peak to the end of lactation. Several mathematical models have been proposed to describe the lactation curve (Nelder 1966; Wood 1967; Yadav et al. 1977; Schaeffer et al. 1977). The parameters of these curves have commonly been estimated by simple regression techniques that generally ignore the covariances among test-day yields.

The parameters of these models contain a genetic component which suggest that the shape of lactation curves can be manipulated by selection. Schneeberger (1978, 1981) and Ferris (1981) reported heritabilities of 0.07–0.14 for the parameters of the gamma function. The relative efficiency of selecting to change the shape of the lactation curve compared to selecting on yield alone has not been reported.

The objectives of this study were (1) To compare three models for test-day yields using five different variance-covariance matrices for each model; (2) To estimate the genetic variances and covariances among the parameters of each of the three models with 305-day milk yield; and (3) To compute relative efficiencies of selection to change the shape of the lactation curve.

MATERIALS AND METHODS

Data

Data set 1 consisted of daily morning and evening milk yields on 1590 Holstein-Friesian cows collected between 1964 and 1973 by Agriculture Canada's Record of Performance (ROP) Program. Only cows with yields through 305 d were included giving 1006 lactations on 775 cows. The morning and evening weights were added together and the data were used to compute variances and covariances among daily milk yields and among inverse yields for the inverse quadratic polynomial model.

In order to obtain estimates of genetic parameters, a larger data set was required. Data set 2 consisted of monthly test-day yields from 102 540 lactations of 73 717 cows collected from 1977 to 1984 on the ROP program. Each cow was required to have a first lactation in this data set, and hence not every cow had an opportunity to have second or later lactations within the period studied. Edits were also made to delete cows with long calving intervals. Later lactations on the same cow had to be made in the same herd as the first lactation. Within a lactation a cow was required to have at least six test-day weighings in order to estimate the parameters of each model, and only the first 12 test-day weighings were used on any cow. Daughters of 4079 sires were represented in data set 2.

Models

Three models describing daily milk yields were studied. Wood's (1967) gamma function has the form

$$y_t = a\, t^b e^{-ct} \tag{1}$$

where y_t is the daily yield on day t; a is a parameter associated with peak yield; b represents

the increasing slope; and c represents the decreasing slope.

To estimate the parameters of this model, natural logarithms are taken of both sides of Eq. 1 giving

$$\ln y_t = \ln a + b \ln t - ct + \epsilon_t \qquad (2)$$

where ϵ_t is a residual error.

The inverse quadratic polynomial model of Nelder (1966) as applied to dairy cattle by Yadav et al. (1977) is written

$$y_t^{-1} = \beta_1 + \beta_0 t^{-1} + \beta_2 t + \epsilon_t \qquad (3)$$

where y_t^{-1} is the inverse daily yield on day t^r; β_1 represents the peak of the lactation curve; β_0 represents the increasing slope; β_2 represents the decreasing slope; and ϵ_t is a residual error.

A regression model was studied with the form

$$y_t = p_0 + p_1\gamma_t + p_2\gamma_t^2 + p_3 w_t + P_4 w_t^2 + e_t \quad (4)$$

where $\gamma_t = t/305$, $w_t = \ln(305/t)$, t = days since calving or days in milk, p_0, p_1, p_2, p_3 and p_4 are the regression coefficients where p_0 is associated with peak yield, p_3 and p_4 are associated with the increasing slope of the curve and p_1 and p_2 are associated with the decreasing slope; and e_t = a residual error for this model.

All three models may be written in matrix notation as:

$$\mathbf{z} = \boldsymbol{Xd} + \boldsymbol{e} \qquad (5)$$

where $\mathbf{z}$ is the vector of observations on one cow, either y_t, $\ln y_t$, or inverse yield; $\mathbf{d}$ is the vector of unknown parameters to be estimated, $\mathbf{X}$ is the matrix of covariates; and $\mathbf{e}$ is the vector of residual errors for that model.

For example, if the gamma function were the model being considered, then

$$\mathbf{d}' = [\ln a \quad b \quad c],$$

and a row of $\mathbf{X}$ would contain

$$[1 \quad \ln t \quad -t],$$

and $\mathbf{z}$ would contain $\ln y_t$.

Usually $V(\mathbf{e}) = V(\mathbf{z})$ is assumed to be an identity matrix times a constant, but can be a general matrix, say $\mathbf{V}$, so that the generalized least squares estimate of $\mathbf{d}$ for a given cow and lactation would be

$$\hat{\mathbf{d}} = (\mathbf{X}'\mathbf{V}^{-1}\mathbf{X})^{-1}\mathbf{X}'\mathbf{V}^{-1}\mathbf{z}. \qquad (6)$$

Five $\mathbf{V}$ matrices were utilized for each model as follows:

(1) $\mathbf{V}_1 = I\sigma^2$,

(2) $\mathbf{V}_2 = V(z)$,

(3) $\mathbf{V}_3 = \mathrm{diag}[V(z)]$, i.e. just the diagonals of V_2 were used,

(4) $\mathbf{V}_4 = V(z-\hat{z})$ where $\hat{z} = \boldsymbol{X}\hat{\boldsymbol{d}}_1$ and $\hat{\boldsymbol{d}}_1 = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'z$, and

(5) $\mathbf{V}_5 = \mathrm{diag}[V(z - \hat{z})]$.

Each $\mathbf{V}$ was of order 305 estimated from the daily milk weights on 1006 cows. Some of the values of these matrices are given in Tables 1 and 2 for selected days within the lactation.

Two criteria were used to compare models: (1) the correlation between actual and predicted daily yields within a lactation of a cow, averaged over cows, and
(2) the percentage squared bias (PSB) computed as

$$PSB = 100\ (\mathbf{y} - \hat{\mathbf{y}})'(\mathbf{y} - \hat{\mathbf{y}})/(\mathbf{y}'\mathbf{y})$$

within a lactation where $\mathbf{y}$ and $\hat{\mathbf{y}}$ refer to actual and predicted daily yields.

These criteria emphasize the agreement between the shapes of the actual and predicted lactation curves rather than simply the area under the curve or the agreement between actual and predicted 305-d yield. If the models are to be used to detect erroneous test-day results for cows, then the fit of daily yields throughout the lactation is more important than the accuracy of predicted 305-d yields.

Similar analyses were applied to the monthly test-day records of 73 717 cows using the five variance-covariance matrices calculated from data set 1. For example, if a cow had tests on days 15, 34, 62, 100 and 150, then the corresponding variances and covariances for those particular days were extracted from $\mathbf{V}_i$ and utilized to estimate $\mathbf{d}_i$ for that cow and model.

Genetic correlations among the parameters of each model with 305-d milk production, by lactation number, were estimated by a maximum likelihood multiple trait analysis as described by Schaeffer (1986). The parameters of each model were obtained using $\mathbf{V}_1 = \mathbf{I}\sigma^2$ only. For example, the traits for the gamma function were $\ln a$, b, c, and 305-d milk yield, for each lactation. The effects of herd-year-season, age and month of calving, and sire were included in the multiple trait analyses.

Selection index theory was used to compute correlated responses in 305-d milk production due to selection on various combinations of parameters

Table 1. Residual and phenotypic variances ($\times 10^2$) and correlations for selected test-days for $\ln y_t$. Variances are on the diagonals with the upper values being the residual variances and the lower being the phenotypic variances. Residual correlations are above the diagonal and phenotypic correlations are below

Test day	15	35	45	90	210	270
15	0.81 6.19	0.10	0.08	−0.16	−0.04	0.09
35	0.84	1.14 6.37	0.46	−0.03	−0.29	0.19
45	0.83	0.91	0.95 6.32	0.04	−0.34	0.20
90	0.72	0.81	0.85	0.77 6.39	−0.23	0.10
210	0.47	0.53	0.58	0.68	1.15 5.59	−0.28
270	0.24	0.29	0.32	0.40	0.67	1.15 8.76

Table 2. Residual and phenotypic variances ($\times 10^5$) and correlations for selected test-days for inverse yields. Variances are on the diagonals with the upper value being the residual variances and the lower being the phenotypic variances. Residual correlations are above the diagonal and phenotypic correlations are below

Test day	15	35	45	90	210	270
15	1.45 2.56	0.66	0.63	−0.13	−0.61	0.19
35	0.80	1.03 2.22	0.74	−0.06	−0.65	0.18
45	0.80	0.89	0.77 2.20	0.02	−0.67	0.16
90	0.68	0.78	0.83	0.44 2.88	−0.16	−0.04
210	0.43	0.50	0.55	0.65	2.15 4.66	−0.18
270	0.20	0.25	0.29	0.34	0.60	2.50 13.59

Table 3. Percentage squared bias (PSB) and correlations (C) between actual and predicted daily yields averaged over 1006 cows for various models and assumed variance-covariance structures

Variance-covariance structure†	Gamma function		Models IQP		Regression	
	PSB	C	PSB	C	PSB	C
$\mathbf{I}\sigma^2$	1.00	0.89	1506.62	−0.86	0.68	0.92
$\mathbf{V}(\mathbf{z})$	3.32	0.81	4.76	0.55	2.58	0.84
diag$[\mathbf{V}(\mathbf{z})]$	0.97	0.89	1.92	0.82	0.69	0.92
$\mathbf{V}(\mathbf{z} - \hat{\mathbf{z}})$	1.00	0.89	1505.92	−0.86	0.68	0.92
diag$[\mathbf{V}(\mathbf{z} - \hat{\mathbf{z}})]$	1.00	0.89	2.22	0.82	0.69	0.92

†$\mathbf{V}(\mathbf{z})$ = phenotypic variance-covariance matrix of $\ln y_t$ for the gamma function, of inverse y_t for the IQP model, and of y_t for the regression model,
diag[ ] is a diagonal matrix composed of the diagonal elements of the matrix within brackets,
$\mathbf{V}(\mathbf{z} - \hat{\mathbf{z}})$ = the variance-covariance matrix of residuals obtained from the respective models assuming $\mathbf{I}\sigma^2$.

Table 4. Average estimates of parameters of three models (using $I\sigma^2$) for age by season subclasses

Model parameters†	(January to June)				(July to December)			
	2	3	4	≥5	2	3	4	≥5
Gamma function								
ln a	2.72	2.84	2.97	3.01	2.73	2.98	3.05	2.97
b (× 10^2)	14.25	19.30	19.50	20.29	12.35	14.13	15.56	19.84
c (× 10^4)	36.68	52.29	58.82	59.36	30.69	45.12	48.39	54.79
IQP model								
β_1 (× 10^2)	3.66	2.43	1.74	1.57	4.01	2.56	2.20	1.83
β_0 (× 10^2)	6.26	6.66	6.44	6.59	7.41	5.34	4.70	6.13
β_2 (× 10^4)	1.52	1.96	2.36	2.23	1.28	1.81	1.81	1.96
Regression model								
p_0	24.98	32.33	43.06	57.17	26.39	30.76	34.12	37.56
p_1	−14.58	−21.18	−49.16	−66.01	−21.08	−25.05	−23.78	−26.54
p_2	0.96	−1.65	6.66	19.93	8.10	5.82	0.91	−0.10
p_3	0.56	0.20	−3.55	−8.84	−0.02	1.43	0.66	−0.29
p_4	−0.46	−0.47	−0.02	0.49	−0.42	−0.67	−0.52	−0.46

†The models are, with t equal to the day within a lactation,
Gamma function: $y_t = at^b e^{-ct}$
IQP model: $y_t^{-1} = \beta_1 + \beta_0 t^{-1} + \beta_2 t + \epsilon_t$
Regression model: $y_t = p_0 + p_1\gamma_t + p_2\gamma_t^2 + p_3 w_t + p_4 w_t^2 + e_t$
where $\gamma_t = t/305$, and $w_t = \ln(305/t)$.

of each model using the estimated genetic correlations and heritabilities from the multiple trait analyses. Results were expressed relative to selection on milk yield alone.

RESULTS AND DISCUSSION

Daily Yield Data

The three models gave essentially unbiased results as the average difference between actual and predicted daily yields averaged within a lactation and over cows was close to zero. The percentage squared bias, however, favored the regression model over the other two models (Table 3). The relationship betwen percentage squared bias and the correlation between actual and predicted daily yields was nearly perfect and therefore led to the same conclusions.

Accounting for covariances among daily yields did not improve the ability of the models to predict daily yields. Except for the IQP model, ignoring the covariances, $\mathbf{V}_1$, was better than the other four matrices and this would simplify the computations for each model as well. The IQP model for many cows gave large negative predictions of daily yield in the early part of the lactation. The IQP model performed better when covariances among yields were included.

Average estimates of the parameters, over cows, for each model when $\mathbf{V}_1$ was used are shown in Table 4 for age-season subclasses. To illustrate the use of these figures, an average 4-yr-old-cow calving in January to June would have a predicted daily yield on day 90 of the lactation of

$$\begin{aligned} \ln y_{90} &= \ln a + \hat{b}\ln(90) - \hat{c}(90) \\ &= 2.97 + 0.1950 \ln(90) - 0.005882\ (90) \\ &= 3.3181, \end{aligned}$$

or

$$\hat{y}_{90} = \exp(3.3181) = 27.61 \text{ kg}$$

for the gamma function,

$$\begin{aligned} \hat{y}_{90}^{-1} &= \hat{b}_1 + \hat{b}_0\ (1/90) + \hat{b}_2(90) \\ &= 0.0174 + 0.0644\ (1/90) + 0.000236\ (90) \\ &= 0.03936, \end{aligned}$$

or

$$\hat{y}_{90} = (0.03936)^{-1} = 25.41 \text{ kg}$$

Table 5. Correlations between actual and predicted monthly test-day yields by lactation number within model for the various variance-covariance structures

Variance-covariance structure†	Gamma function			Models IQP			Regression		
	1st	2nd	3rd	1st	2nd	3rd	1st	2nd	3rd
$\mathbf{I}\sigma^2$	0.88	0.94	0.94	0.87	0.91	0.92	0.95	0.98	0.98
$\mathbf{V}(\mathbf{z})$	0.86	0.93	0.94	0.77	0.85	0.89			
diag$[\mathbf{V}(\mathbf{z})]$	0.88	0.94	0.94	0.84	0.89	0.91			
$\mathbf{V}(\mathbf{z} - \hat{\mathbf{z}})$	0.87	0.94	0.94	0.84	0.89	0.91			
diag$[\mathbf{V}(\mathbf{z} - \hat{\mathbf{z}})]$	0.88	0.94	0.94	0.84	0.90	0.91			

†$\mathbf{V}(\mathbf{z})$ = phenotypic variance-covariance matrix of ln y_t for the gamma function, of inverse y_t for the IQP model, and of y_t for the regression model.
diag[ ] is a diagonal matrix composed of the diagonal elements of the matrix within brackets.
$\mathbf{V}(\mathbf{z} - \hat{\mathbf{z}})$ = the variance-covariance matrix of residuals obtained from the respective models assuming $\mathbf{I}\sigma^2$.

for the IQP model, and

$$\begin{aligned}\hat{y}_{90} &= \hat{p}_0 + \hat{p}_1(90/305) + \hat{p}_2(90/305)^2 + \hat{p}_3\ln(305/90) + \hat{p}_4[\ln(305/90)]^2 \\ &= 43.06 - 40.16(0.295) + 6.66(0.295)^2 + 3.55(1.2205) - 0.02(1.2205)^2 \\ &= 27.43 \text{ kg for the regression model.}\end{aligned}$$

Monthly Test Data

Data on monthly test-day weighings of 73 717 cows were also used to compare the models within first, second, and third lactations (Table 5). The regression model was applied using only $\mathbf{V}_1$. The results were similar to those on the daily milk weights except that the IQP model does not appear to be as poor as in Table 3. Accounting for covariances among test-day yields does not seem to offer any advantages in the prediction of daily yields. The regression model performed the best, but requires at least six test weighings in order to estimate the parameters of the model. This could be the main disadvantage of the regression model, in practice, if used for extending part lactation records.

Genetic Correlations with 305-d Milk Yield

A maximum likelihood multiple trait analysis was used to estimate sire and residual variance-covariance matrices for the parameters of each model and 305-d milk yield. The heritabilities of the parameters are presented in Table 6 for each model. The heritability of milk yield differed slightly depending on the other traits which were included in the multiple trait analysis. The heritabilities of nearly all parameters changed with lactation number, without any apparent trends.

Table 6. Heritabilities of model parameters for the first two lactations

Model parameters†	Lactation	
	1st	2nd
Gamma function		
ln a	0.11	0.12
b	0.30	0.17
c	0.23	0.23
305-d milk	0.28	0.25
IQP Model		
β_1	0.52	0.90
β_0	0.01	0.04
β_2	0.45	0.00
305-d milk	0.31	0.25
Regression Model		
p_0	0.04	0.16
p_1	0.07	0.30
p_2	0.08	0.35
p_3	0.15	0.47
p_4	0.16	0.47
305-d milk	0.30	0.36

†The models are, with t equal to the day within a lactation,
Gamma function: $y_t = a\, t^b\, e^{-ct}$
IQP model: $y_t^{-1} = \beta_1 + \beta_0 t^{-1} + \beta_2 t + \epsilon_t$
Regression model: $y_t = p_0 + p_1\gamma t + p_2\gamma_t^2 + p_3 w_t + p_4 w_t^2 + e_t$
where $\gamma_t = t/305$ and $w_t = \ln(305/t)$.

Estimates of genetic correlations between the same parameters in different lactations

Table 7. Genetic correlations between the same parameters in first and second lactations and with 305-d milk yield

Model parameter†	Between same parameters	With 305-d milk	
		1st	2nd
Gamma function			
ln a	0.06	0.12	0.07
b	0.09	0.00	0.02
c	0.12	−0.09	−0.11
305-d milk	0.37		
IQP model			
β_1	0.22	−0.15	−0.11
β_0	0.05	−0.05	−0.17
β_2	−0.01	0.02	0.03
305-d milk	0.30		
Regression model			
p_0	0.00	0.01	0.00
p_1	0.00	−0.02	−0.01
p_2	0.00	0.02	0.00
p_3	0.01	−0.03	−0.03
p_4	0.00	0.03	0.02
305-d milk	0.17		

†The models are, with t equal to the day within a lactation,
Gamma function: $y_t = a\ t^b\ e^{-ct}$
IQP model: $y_t^{-1} = \beta_1 + \beta_0 t^{-1} + \beta_2 t + \epsilon_t$
Regression model: $y_t = p_0 + p_1\gamma_t + p_2\gamma_t^2 + p_3 w_t + p_4 w_t^2 + e_t$
where $\gamma_t = t/305$, and $w_t = \ln(305/t)$.

were very low (Table 7). The genetic correlation between milk yield in first and second lactations ranged between 0.17 and 0.37, but values of 0.92 or greater would have been expected (Tong et al. 1979). The low estimates may be due to the fact that only 37.6% of the cows had a second lactation compared to a more normal rate of 75%. Not all cows had an opportunity to complete a second lactation. The other correlations could also be low for the same reason.

The parameters of each of the three models had very small genetic correlations with 305-d milk yield, especially the parameters of the regression model. In general, the parameters associated with peak yield (ln a and β_1) were positively correlated with 305-d yield while the slope parameters had correlations near zero or negative with 305-d yield. In the gamma function, the c parameter was negatively correlated with 305-d yield, but as c becomes smaller the downward slope of the lactation curve becomes less, meaning the

Table 8. Relative efficiencies of selection on parameters of models and 305-d milk yield (MILK) in first lactation

Selection on	Relative efficiency (%)
MILK	100.0
Gamma function	
MILK, ln a	74.7
MILK, c	74.0
MILK, b	70.8
MILK, ln a, b, c	44.1
Regression model	
MILK, p_3, p_4	80.9
MILK, p_3	71.9
MILK, p_1	71.0
MILK, p_0	70.9
MILK, p_1, p_4	70.3
MILK, p_2	70.1
MILK, p_4	69.6
MILK, p_0, p_1, p_2, p_3, p_4	52.4

The models are, with t equal to the day within a lactation,
Gamma function: $y_t = a\ t^b\ e^{-ct}$
Regression model: $y_t = p_0 + p_1\gamma t + p_2\gamma_t^2 + p_3 w_t + p_4 w_t^2 + e_t$
where $\gamma_t + t/305$, and $w_t = \ln(305/t)$.

cow is more persistent. Large values of ln a and b were desirable in increasing 305-d yield.

For the IQP model, values of β_0 and β_2 that were near zero or negative would be advantageous in increasing 305-d yield while β_1 should be small and positive, but never zero or negative. Hence, the negative genetic correlation estimates of β_1 and β_0 with 305-d yield could be anticipated.

In the regression model, positive increasing values of all parameters lead to greater 305-d yields. However, except for p_0, the other parameter estimates tend to be negative for most cows. None of the parameters appeared to have any genetic relationship with 305-d yield. Hence the regression model could be used to change the shape of the lactation curve without adversely changing 305-d yield. The parameters of the regression model were strongly correlated with each other with genetic correlation estimates of -0.38 (p_1 with p_2), 0.46 (p_1 with p_3), -0.50 (p_1 with p_4), -0.36 (p_2 with p_3), 0.40 (p_2 with p_4), and -0.73 (p_3 with p_4). Selection on p_1 would decrease p_2 and p_4, but would increase p_0 and p_3.

Correlated Responses

Various selection schemes were examined involving the parameters of the models and 305-d milk yield. The correlated response to selection for milk yield was compared to the response from selecting on milk yield alone (Table 8). Only relative efficiencies of 70% or greater were presented in Table 8 including three special cases in which efficiency was less than 70%. The best index incorporated 305-d milk yield and parameters p_3 and p_4 of the regression model. The parameters p_3 and p_4 have their greatest impact on the early part of the lactation curve since $\ln(305/t)$ decreases as t increases. None of the indexes, without 305-d yield included, gave a relative efficiency greater than 13%.

In conclusion, accounting for covariances among test day yields for milk did not lead to more accurate predictions of daily yields. The regression model considered in this study was slightly superior to Wood's gamma function and both of these methods were superior to the inverse quadratic polynomial model. Selection to change the shape of the lactation curve could reduce the relative effectiveness of improving 305-d yield by 20% or more. An index containing 305-d milk yield and parameters related to the peak of the lactation curve and the slope from calving to the peak seems to offer the most promise.

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